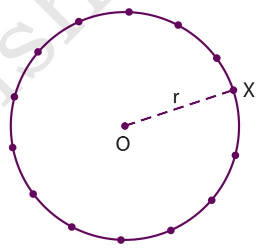
6 Mensuration

M11CH6

6.1. CIRCLE AS A LOCUS

A **locus** is a set of all the points whose position is defined by a particular rule, law or condition.

A circle is the **locus** of a point which moves in a plane in such a way that its distance from a fixed point in the plane is constant. Thus, a circle is the set of all points in a plane which are at a given constant distance (equidistant) from a fixed point in the plane.



(In the phrase 'in a plane' is omitted, we get the definition of a sphere.)

The fixed point is called the **centre** and the constant distance from the fixed point *i.e.*, centre is called the **radius** of the circle.

A circle with centre O and radius r is usually denoted by C(O, r) as shown in figure. Therefore,

$$C\,(O, r) = \{X : OX = r\}$$

Fix a point O in the plane (your notebook). Plot a number of points at a given distance, say 5 cm, from O. Join these points by a free hand curve. This curve has the property that a point moving along it is at a distance 5 cm from the fixed point O and conversely, all points in the plane at a distance 5 cm from O lie on this curve. This curve is the circle with centre O and radius 5 cm.

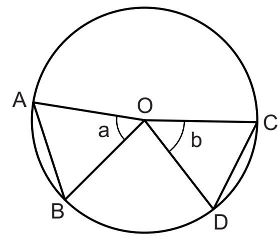
> **Note:** In practice we use a compass with a well sharpened pencil at one end to draw circles.

6.2. CIRCLE THEOREMS

Theorem 1: *Equal chords or arcs of a circle subtend equal/same angles at the centre of a circle.*

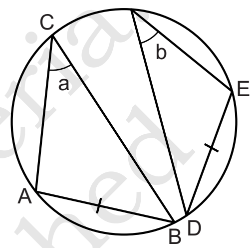
Conversely, if the angles subtended by the chords or arcs of a circle at the centre are equal, then chords or arcs are equal.

Thus, AB = CD $\Leftrightarrow$ $\angle$AOB = $\angle$COD $\Leftrightarrow$ $a = b$



Theorem 2: *Equal arcs or chords subtend the same angles at the circumference of a circle.*

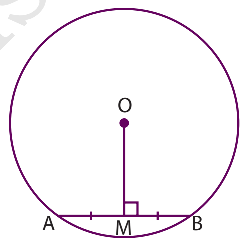
Thus, AB = DE $\Leftrightarrow$ $\angle$ACB = $\angle$DEF $\Leftrightarrow$ $a = b$



Theorem 3: *The perpendicular from the centre of a circle to a chord bisects the chord.*

Conversely, the straight line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

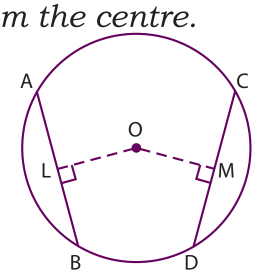
Thus, OM $\perp$ AB $\Leftrightarrow$ M is mid-point of AB.



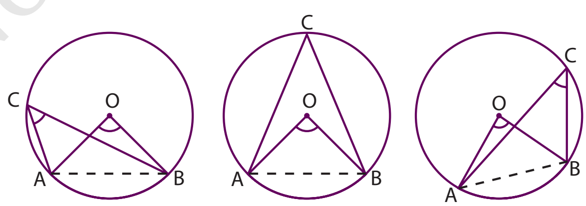
Theorem 4: *Equal chords of a circle are equidistant from the centre.*

Conversely, chords of a circle equidistant from the centre are equal.

Thus, AB = CD $\Leftrightarrow$ OL = OM



Theorem 5: *The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.*



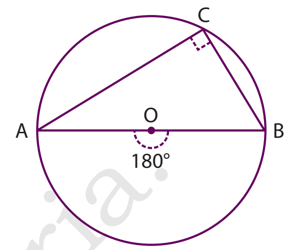
Thus, if arc AB subtends $\angle$AOB at the centre and $\angle$ACB at any point C on the remaining part of the circle, then

$$\angle AOB = 2\,\angle ACB \quad \text{or} \quad \angle ACB = \frac{1}{2}\,\angle AOB$$

Theorem 6: *The angle subtended by a diameter of a circle at the circumference is a right angle (= 90°).*

Or

An angle in a semicircle is a right angle.



If AB is a diameter, then ∠AOB = 180°.

But $\angle AOB = 2\angle ACB$

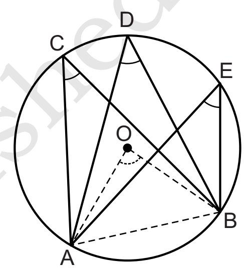
Therefore, $2\angle ACB = 180°$

⇒ $\angle ACB = 90°$

Theorem 7: *Angles in the same segment of a circle are equal.*

Or

The angles a chord or arc substends at the circumference in the same segment of a circle are equal.



Since, $\angle ACB = \frac{1}{2}\angle AOB$

and $\angle ADB = \frac{1}{2}\angle AOB$

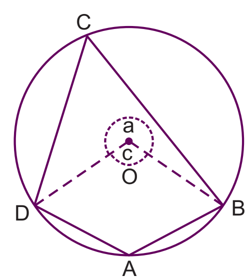
Similarly, $\angle AED = \frac{1}{2}\angle AOB$

Therefore, $\angle ACB = \angle ADB = \angle AEB$

Theorem 8: *The opposite angles of a cyclic quadrilateral are supplementary i.e., they add up to 180°.*

Or

The sum of the angles of a chord or an arc substends at the circumference of opposite segments of a circle is equal to 180°.



(A quadrilateral is called a cyclic quadrilateral if all its vertices lie on a circle.)

Since the angle subtended by an arc at the centre is double the angle it subtends at any point on the remaining circle therefore,

$$\angle a = 2\angle A \text{ and } \angle c = 2\angle C$$

Adding, $\angle a + \angle c = 2(\angle A + \angle C)$

or $\angle A + \angle C = \frac{1}{2}(\angle a + \angle c) = \frac{1}{2}(360°) = 180°$

[∵ Sum of angles at a point is 360°]

Also, the sum of all angles of a quadrilateral is 360°.

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad (\angle A + \angle C) + (\angle B + \angle D) = 360°$

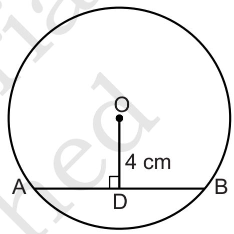
$\Rightarrow \quad 180° + (\angle B + \angle D) = 360°$

$\Rightarrow \quad \angle B + \angle D = 180°$

Hence, ABCD is a cyclic quadrilateral.

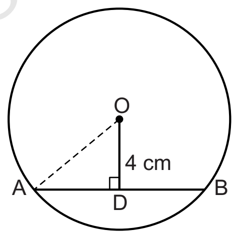
$\Rightarrow \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

Example 1: *In the figure, O is the centre of the circle and AB is the chord. If OD ⊥ AB, then find radius of the circle. Here, AB = 6 cm, OD = 4 cm.*



Solution: **Construction:** Join OA.

Since the perpendicular from the centre of a circle to a chord bisects the chord therefore,



$\therefore \quad AD = DB = \frac{1}{2}AB$ (Theorem 3)

$= \frac{1}{2} \times 6 = 3$ cm

In right angled triangle ODA,

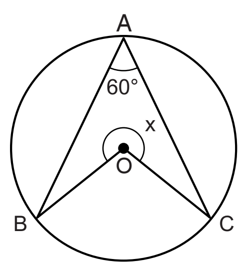
$OA^2 = OD^2 + AD^2$ (By Pythagoras Theorem)

$= 4^2 + 3^2 = 16 + 9 = 25$

$\Rightarrow \quad OA = 5$ cm

Hence, the radius of the circle is 5 cm.

Example 2: *In the figure, if O is the centre of the circle and ∠BAC = 60°, find the value of reflex angle x.*



Solution: Since angle subtended by an arc BC of a circle at the centre of the circle is double the angle subtended by the arc at any point on the remaining part of the circle, therefore

$\angle BOC = 2 \times \angle BAC$ (Theorem 5)

$= 2 \times 60°$

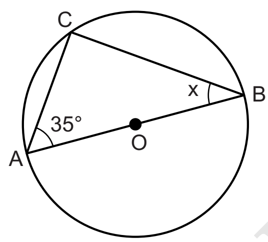
$= 120°$

$\therefore \quad$ Reflex $\angle BOC + \angle BOC = 360°$

(Sum of all angles round a point is 360°)

$$\Rightarrow \quad x + 120° = 360°$$
$$\Rightarrow \quad x = 360° - 120°$$
$$= 240°$$

Example 3: *In the figure, O is the centre of the circle and ∠CAB = 35°. Find the measure of x.*



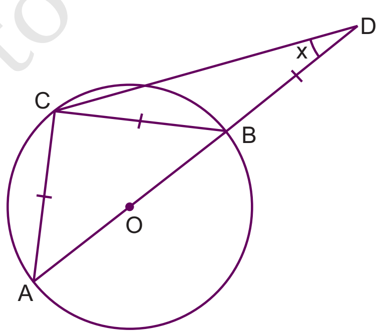
Solution: Since angle in a semicircle is 90°, therefore

$$\angle ACB = 90° \qquad \text{(Theorem 6)}$$

In ΔABC, sum of the interior angles of a triangle is 180° i.e.,

$$\angle CAB + \angle ABC + \angle ACB = 180°$$
$$\Rightarrow \quad 35° + x + 90° = 180°$$
$$\Rightarrow \quad x + 125° = 180°$$
$$\Rightarrow \quad x = 180° - 125°$$
$$\Rightarrow \quad x = 55°$$

Example 4: *In the figure, AC = BC = BD, find x.*



Solution: Here, AOB is a diameter, therefore

$$\angle ACB = 90° \qquad \text{(Angle in a semicircle)}$$

In ΔABC, $\angle BAC + \angle ABC = 90°$

But $\angle BAC = \angle ABC$

(Angles opposite equal sides)

Therefore, $\angle BAC = \angle ABC = 45°$

Since, $\angle ABC + \angle CBD = 180°$ (Linear pair at B)

$\Rightarrow \quad 45° + \angle CBD = 180°$

$\Rightarrow \quad \angle CBD = 180° - 45° = 135°$

In triangle CBD,

$\angle BCD = \angle CDB = x$ (Angles opposite equal sides)

$\therefore \quad \angle BCD + \angle CDB + \angle CBD = 180°$

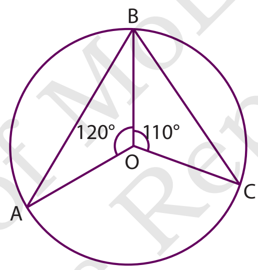
(Sum of angles of a triangle is 180°)

$\Rightarrow \quad x + x + 135° = 180°$

$\Rightarrow \quad 2x = 180° - 135° = 45°$

$\Rightarrow \quad x = \dfrac{45°}{2} = 22.5°$

Example 5: *In the figure, find ∠ABC.*



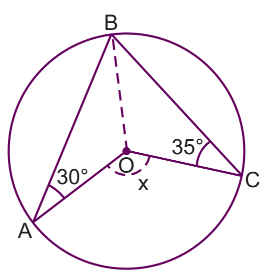
Solution: Reflex $\angle AOC = \angle AOB + \angle COB = 120° + 110° = 230°$

$\Rightarrow \quad$ Obtuse $\angle AOC = 360° - 230° = 130°$ (Angle at a point)

Now, $\angle ABC = \dfrac{1}{2}$(Obtuse $\angle AOC$) (Theorem 5)

$= \dfrac{1}{2}(130°) = 65°$

Example 6: *In the given figure, find x.*



Solution: In triangle AOB,

$$\angle ABO = \angle BAO, \qquad \text{Since } OA = OB \text{ (radius)}$$

$$\Rightarrow \quad \angle ABO = 30^\circ$$

In triangle COB,

$$\angle CBO = \angle BCO, \qquad \text{Since } OB = OC \text{ (radius)}$$

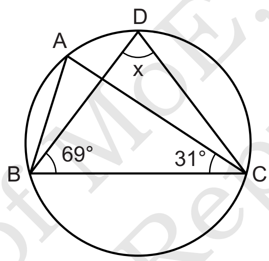
$$\Rightarrow \quad \angle CBO = 35^\circ$$

Now, $\angle ABC = \angle ABO + \angle CBO = 30^\circ + 35^\circ = 65^\circ$

Since angle subtended by an arc of a circle at the centre of the circle is double the angle subtended by the arc at any point on the remaining part of the circle, therefore,

$$x = 2\,\angle ABC = 2 \times 65^\circ = 130^\circ.$$

Example 7: *In the figure, $\angle ABC = 69^\circ$ and $\angle ACB = 31^\circ$. Find the value of x.*



Solution: In triangle ABC, sum of the interior angles of a triangle is 180°, therefore

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$\Rightarrow \quad \angle BAC + 69^\circ + 31^\circ = 180^\circ$$

$$\Rightarrow \quad \angle BAC = 180^\circ - (69^\circ + 31^\circ) = 180^\circ - 100^\circ$$

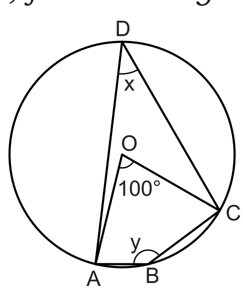
$$\Rightarrow \quad \angle BAC = 80^\circ \qquad \ldots (1)$$

Now, angles in the same segment of a circle are equal.

$$\therefore \quad \angle BDC = \angle BAC \qquad \text{(Theorem 7)}$$

$$\Rightarrow \quad x = 80^\circ \qquad \text{(Using (1))}$$

Example 8: *In the given figure, find x and y.*



Solution: Since arc ABC subtends ∠AOC at the centre of the circle and ∠ADC at a point on the remaining part of the circle, therefore,

$$x = \frac{1}{2}\angle AOC = \frac{1}{2} \times 100° = 50° \quad \text{(Theorem 5)}$$

In cyclic quadrilateral ABCD,

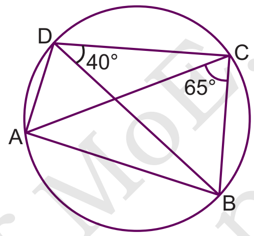
$$x + y = 180° \quad \text{(Theorem 8)}$$

$$\Rightarrow \quad 50° + y = 180°$$

$$\Rightarrow \quad y = 180° - 50° = 130°$$

Hence, $x = 50°$ and $y = 130°$.

Example 9: *In the given figure, find ∠ABC.*



Solution: Arc AB subtends angles ADB and ACB in the same segment.

Therefore, $\angle ADB = \angle ACB = 65°$ (Theorem 7)

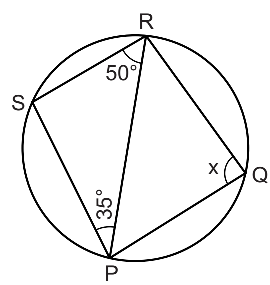
Now, $\angle ADC = \angle ADB + \angle BDC = 65° + 40° = 105°$

Now, in cyclic quadrilateral ABCD,

$$\angle ADC + \angle ABC = 180° \quad \text{(Theorem 8)}$$

$$\Rightarrow \quad 105° + \angle ABC = 180° \Rightarrow \angle ABC = 180° - 105° = 75°$$

Example 10: *In the figure, PQRS is a cyclic quadrilateral. Find the value of x.*



Solution: In Δ PSR, sum of all interior angles of a triangle is 180°.

$\therefore \quad \angle PSR + \angle SPR + \angle SRP = 180°$

$\Rightarrow \quad \angle PSR + 35° + 50° = 180°$

$\Rightarrow \quad \angle PSR + 85° = 180°$

$\Rightarrow \quad \angle PSR = 95°$ ... (1)

$\because$ PQRS is a cyclic quadrilateral, therefore opposite angle of a cyclic quadrilateral are supplementary.

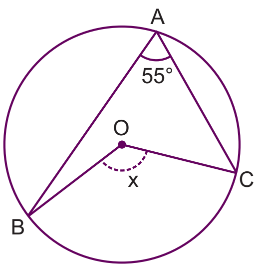
$\therefore \quad \angle PSR + \angle PQR = 180°$ (Theorem 8)

$\Rightarrow \quad 95° + x = 180°$ (From (1))

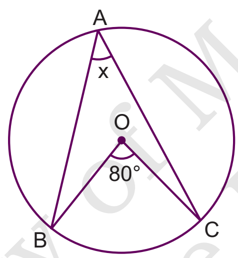
$\Rightarrow \quad x = 180° - 95° = 85°$

EXERCISE 6.1

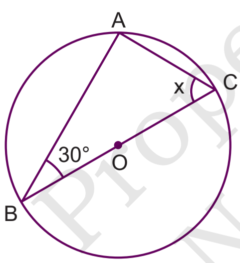
1. In the following figures, O is the centre of the circle. Find x:



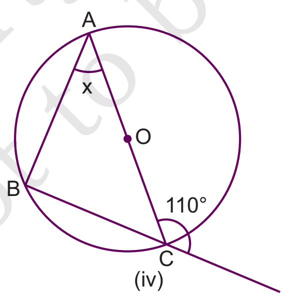
(i)



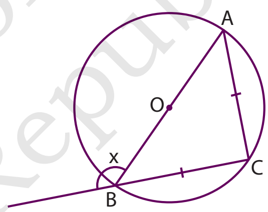
(ii)



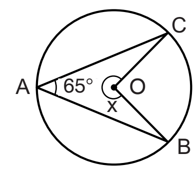
(iii)



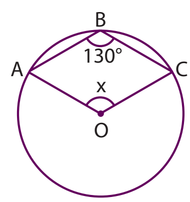
(iv)



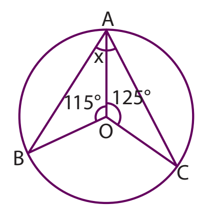
(v)



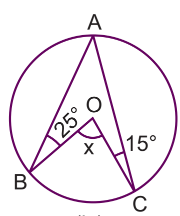
(vi)



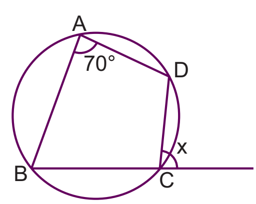
(vii)



(viii)

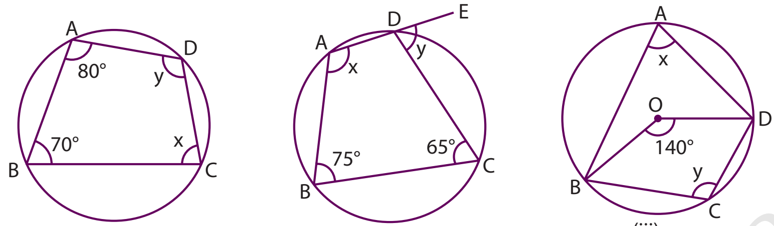


(ix)

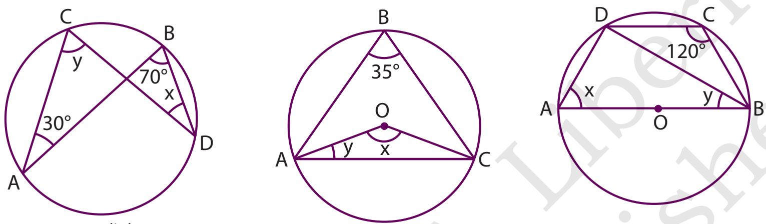


(x)

2. In the following figures, find x and y.

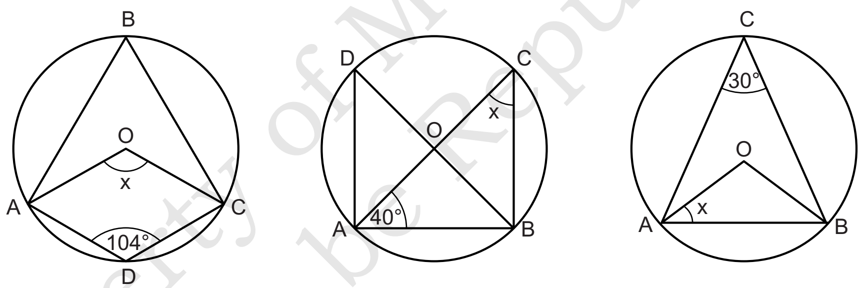


(i) (ii) (iii)



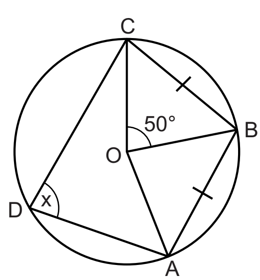
(iv) (v) (vi)

3. In the figures below, O is the centre of the circles. Find the values of x.

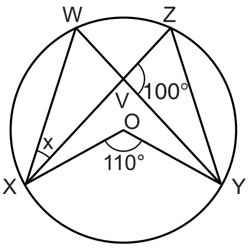


(i) (ii) (iii)

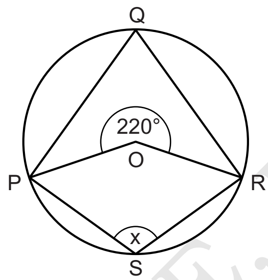
4. In the figure, O is the centre of the circle and AB = CB. Find the value of x.



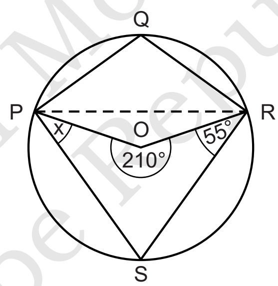
5. In the figure, O is the centre of the circle and XZ and WY intersect at V. If ∠XOY = 110° and ∠YVZ = 100°, find the value of x.



6. In the figure, O is the centre of the circle. Find the value of *x*.

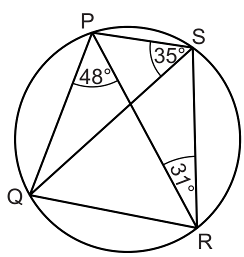


7. In the figure, O is the centre of the circle. Find the value of *x*.

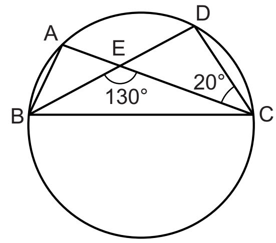


8. In the figure, P, Q, R and S are points on the circle. If ∠QPR = 48°, ∠PSQ = 35° and ∠PRS = 31°, find

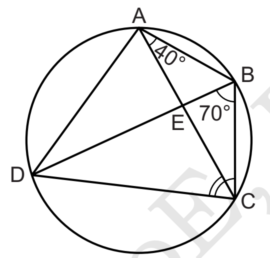
(i) ∠PQR (ii) ∠QRS



9. In the figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.

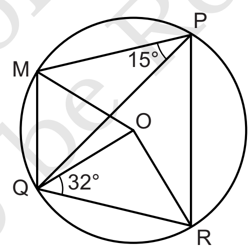


10. ABCD is a cyclic quadrilateral whose diagonal intersect at a point E. If ∠DBC = 70°, ∠BAC = 40°, find ∠BCD. Further, if AB = BC, find ∠ECD.



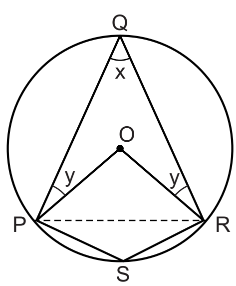
11. In the figure, O is the centre of the circle, ∠OQR = 32° and ∠MPQ = 15°. Find

(i) ∠QPR (ii) ∠MQO



12. In the figure, O is the centre of the circle. Quadrilateral OPSR is a rhombus. Find

(i) x (ii) y (iii) ∠QRS

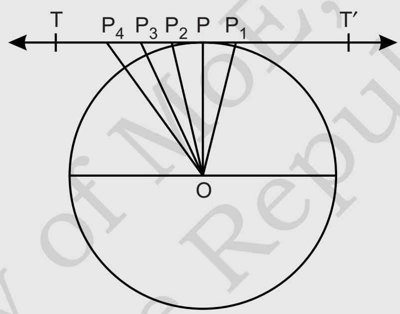


6.3. TANGENTS TO A CIRCLE

Tangent at a Point on the Circle

ACTIVITY 6.1

Using a compass, draw a circle with centre O and any convenient radius on the paper. Draw the tangent TT′ intersecting the circle, at point P. Mark points P_1, P_2, P_3, P_4, etc. on TT′. Join O to P, P_1, P_2, P_3, P_4, etc. Measure OP, OP_1, OP_2, OP_3, OP_4 etc. You will find that OP is the shortest line segment. But you know that the shortest line segment from a point to a line is the perpendicular from the point to the line. It follows that OP is perpendicular to TT′. But OP is the radius at the point of contact of the tangent TT′.

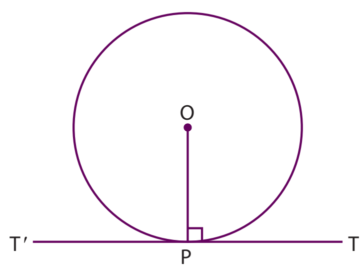


The above activity leads us an important property of the tangent to a circle, to be called **Tangent-Radius property**. It follows that the tangent at a point on a circle and the radius through the point of contact are perpendicular to each other.

Theorem 9: (Tangent-Radius Theorem)

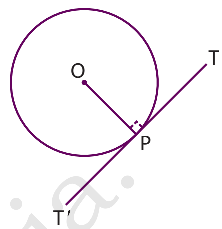
The tangent at any point of a circle and the radius through the point are perpendicular to each other.

Thus, if P is any point on a circle with centre O and T′PT is the tangent at P, then OP ⊥ T′PT (shown in figure).



$\therefore \quad \angle OPT = \angle OPT' = 90°$

The theorem is very useful in constructing the tangent to a given circle at a given point on it.



Consider a circle with centre O. Let P be any point on it. To construct the tangent to the circle at P (See figure):

(i) Join OP.

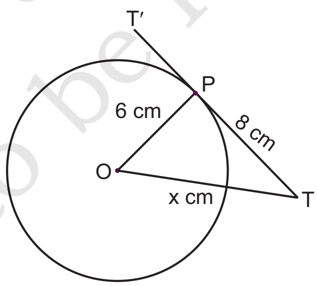
(ii) Draw a line T′PT perpendicular to OP at P.

Then T′PT is the required tangent to the circle at P.

P is called the point of contact.

Since only one perpendicular can be drawn to OP at P, only one tangent can be drawn to a circle at a given point on it. Thus, **the tangent at any point on a circle is unique.** The perpendicular from the centre of a circle to a tangent meets it all the point of contact. The perpendicular to a tangent through its point of contact passes through the centre of the circle.

Example 1: *In the figure, T′PT is the tangent to the circle at P. Find the value of x.*



Solution: Since T′PT is the tangent at P,

$\therefore \quad OP \perp T'PT \quad$ (By Tangent-Radius Theorem)

$\Rightarrow \quad \angle OPT = 90°$

In ΔOPT, $\quad OP^2 + PT^2 = OT^2 \quad$ (Pythagoras Theorem)

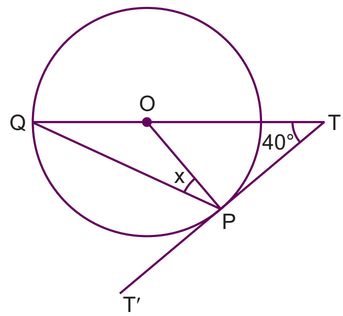
$\Rightarrow \quad 6^2 + 8^2 = x^2$

$\Rightarrow \quad 36 + 64 = x^2$

$\Rightarrow \quad x^2 = 100$

$\Rightarrow \quad x = 10 \text{ cm}$

Example 2: *In the figure, T′PT is the tangent to the circle at P. Find the value of x.*



Solution: Since T′PT is the tangent at P,

Therefore, $OP \perp T'PT$ (By Tangent-Radius Theorem)

$\Rightarrow \angle OPT = 90°$

In triangle OPT, $\angle POT + \angle PTO = 90°$

$\Rightarrow \angle POT + 40° = 90°$

$\Rightarrow \angle POT = 90° - 40° = 50°$

Now, $\angle POT + \angle POQ = 180°$ (Linear pair)

$\Rightarrow 50° + \angle POQ = 180°$

$\Rightarrow \angle POQ = 130°$

In triangle POQ,

$\angle OPQ + \angle OQP + \angle POQ = 180°$

(Sum of the angles of a triangle is 180°)

$\Rightarrow \angle OPQ + \angle OPQ + 130° = 180°$

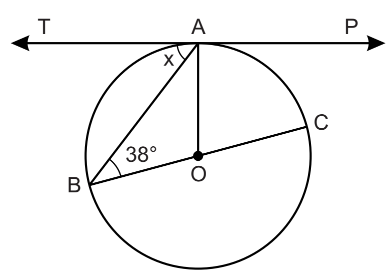
($\angle OPQ = \angle OQP$, angles on equal side)

$\Rightarrow x + x = 180° - 130°$

$\Rightarrow 2x = 50$

$\Rightarrow x = 25°$

Example 3: *In the figure, BC is a diameter of the circle with centre O and PAT is the tangent at A. If ∠ABC = 38°, find the value of x.*



Solution: Since PAT is the tangent at P,

$\therefore OA \perp PAT$

$\Rightarrow \angle OAT = 90°$ (By Tangent-Radius Theorem)

In triangle OAB,

$\Rightarrow$ $OA = OB$ (Radius of circle)

$\Rightarrow$ $\angle OAB = \angle OBA$ (Angles opposite to equal side of a triangle are equal)

$= 38°$

$\therefore$ $\angle OAT = \angle BAT + \angle OAB$

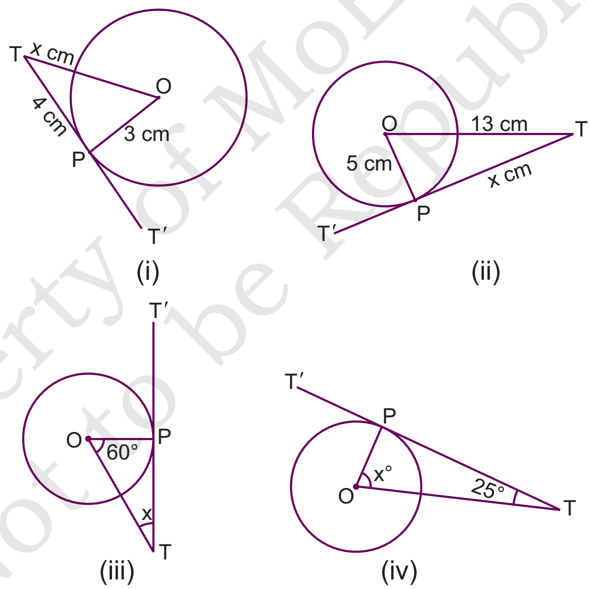
$\Rightarrow$ $90° = x + 38°$

$\Rightarrow$ $x = 90° - 38° = 52°$

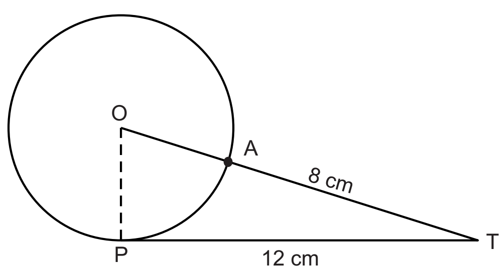
EXERCISE 6.2

1. Draw a circle with centre O and radius 3 cm. Take any point P on it. Draw the tangent to the circle at P.

2. In each of the following figures, find x. (T′PT is the tangent at P.)



3. In the figure, PT is a tangent to a circle of centre O. Find the length OA.

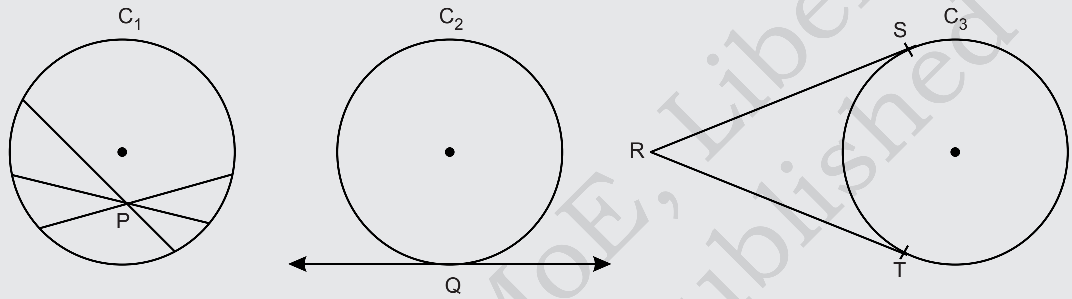


Tangent From an External Point

To understand the notion of the number of tangents to a circle from a point lying outside it, we can perform a very simple activity as follows:

ACTIVITY 6.2

Draw three separate circles C_1, C_2 and C_3 in the plane of the book and then mark a point P inside C_1, a point Q on C_2 and a point R outside C_3 (as shown in figures (i), (ii) and (iii)).



(i) (ii) (iii)

Observations of Activity

Case (I): When the point P lies inside the circle C_1, we find that all the lines passing through P intersect C_1 in two points. So, by the definition of a tangent, none of these can be a tangent to circle C_1.

Case (II): When the point Q lies on the circle C_2, only one line can be drawn through Q which intersects C_2 in exactly one point. So, by definition, this line is the tangent to the circle C_2. In this case, Q itself is the point of contact of the tangent drawn at this point.

Case (III): Lastly, when the point R lies outside the circle, then exactly two tangents RS and RT can be drawn to the circle C_3 from the point R. In this case, S and T are the points of contact of the tangents RS and RT.

From the observations, of above Activity, we are in a position to draw the conclusions as follows:

In a circle C(O, *r*) if a point P is

(i) Inside a circle, no tangent can be drawn through a point lying inside a circle.

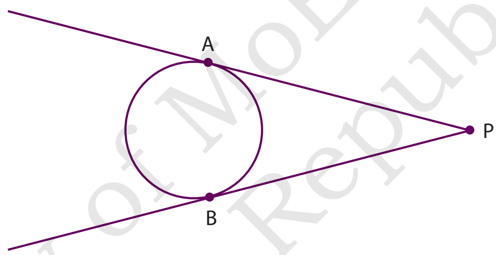
(ii) On the circle, one and only one tangent can be drawn through a point lying on a circle.

(iii) Outside the circle, there are exactly two tangents drawn to circle through a point lying outside it.

Theorem 10: (Equal Tangent-Length Theorem)

If two tangents are drawn from an external point to a circle, then the tangents are equal in length.

Thus, if P is an external point to a circle and PA, PB are two tangents drawn from a point P to the circle, A and B being the points of contact, then PA = PB (See figure).



Notes:

Consider a circle with centre O and radius *r*. Let P be an external point. Join OP. Let PA and PB be the two tangents from P to the circle. Since they are equal, let PA = PB. Then

1. OA = OB = *r* (Radius of circle)

2. ∠OAP = ∠OBP = 90° (By Tangent-Radius Theorem)

3. Triangles OAP and OBP are right angled triangles at A and B respectively.

4. ∠OPA = ∠OPB (Tangents PA and PB are equally inclined to OP)

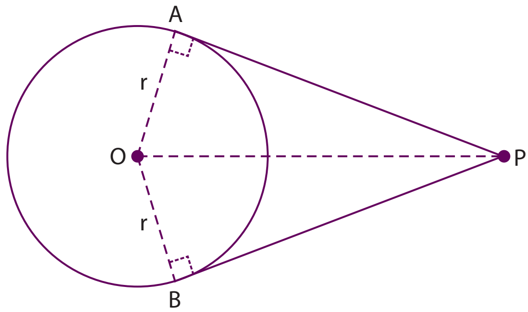
5. ∠POA = ∠POB (Tangents PA and PB subtend equal angles at the centre O)

6. In two right angled trianges OAP and OBP, by Pythagoras theorem,

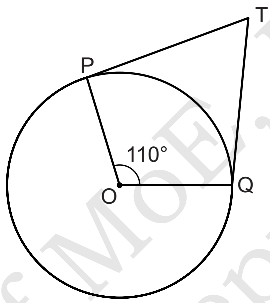
$$PA^2 = OA^2 + OP^2$$

and

$$PB^2 = OB^2 + OP^2$$



Example 4: *In the figure, if TP an TQ are two tangents to a circle with centre O so that ∠POQ = 110°. Find the measure of ∠PTQ.*



Solution: Since TP and TQ are the tangents at P and Q respectively, therefore

$$\angle OPT = \angle OQT = 90° \text{ (By Tangent-Radius Theorem)}$$

In quadrilateral OPTQ,

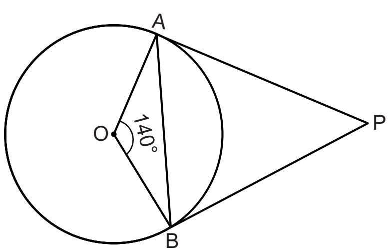
Sum of all the four angles = 360°

$\Rightarrow$ 110° + 90° + 90° + ∠PTQ = 360°

$\Rightarrow$ 290° + ∠PTQ = 360°

$\Rightarrow$ ∠PTQ = 360° – 290° = 70°

Example 5: *In the figure, tangents PA and PB to a circle with centre 0 are drawn from a point P outside the circle. If ∠AOB = 140° and AB is joined, find*



(i) $\angle OAB$ (ii) $\angle APB$ (iii) $\angle ABP$

Solution:

(i) In triangle OAB,

$$OA = OB \quad \text{(Radii of same circle)}$$

$\Rightarrow$ $\angle OAB = \angle OBA = \theta$ (say)

By angle sum property of a triangle,

$\Rightarrow$ $\theta + \theta + 140° = 180°$

$\Rightarrow$ $2\theta = 180° - 140° = 40°$

$\theta = 20° \Rightarrow \angle OAB = 20°$

(ii) $\angle OAP = \angle OBP = 90°$ (By Tangent-Radius Theorem)

In quadrilateral OAPB,

$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

$\Rightarrow$ $90° + \angle APB + 90° + 140° = 360°$

$\Rightarrow$ $\angle APB + 320° = 360°$

$\Rightarrow$ $\angle APB = 360° - 320° = 40°$

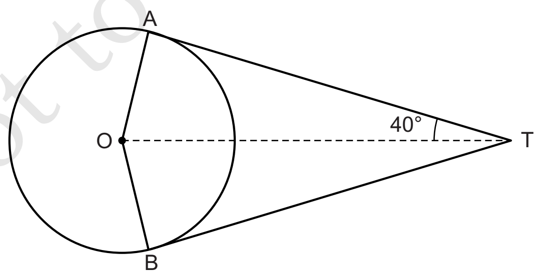
(iii) $\because$ $\angle OBP = 90°$

$\angle OBA + \angle ABP = 90°$

$\Rightarrow$ $20° + \angle ABP = 90°$

$\Rightarrow$ $\angle ABP = 90° - 20° = 70°$

Example 6: *In the figure, $\angle ATO = 40°$. Find the value of $\angle AOB$.*



Solution: $\because$ TA and TB are two tangents of the circle from the external point T.

$\because$ TA and TB are equally inclined to OT.

$\Rightarrow$ $\angle ATO = \angle BTO = 40°$

$\Rightarrow$ $\angle ATB = 2 \times \angle ATO = 2 \times 40° = 80°$ ...(1)

Also, OA ⊥ AT and OB ⊥ BT

$\therefore \quad \angle OAT = \angle OBT = 90°$ ...(2)

(By Tangent-Radius Theorem)

In quadrilateral OATB,

$\angle AOB + \angle OAT + \angle ATB + \angle OBT = 360°$

(Angle-Sum Property of a Quadrilateral)

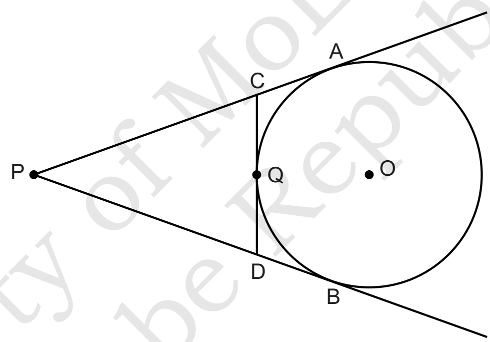
$\Rightarrow \quad \angle AOB + 90° + 80° + 90° = 360°$ (Using (1) and (2))

$\Rightarrow \quad \angle AOB + 260° = 360°$

$\Rightarrow \quad \angle AOB = 360° - 260°$

$\Rightarrow \quad \angle OAB = 100°$

Example 7: *In the figure, PA and PB are two tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length of PC?*



Solution: Since PA and PB are two tangents of circle from the external point P.

PA = PB (By Equal Tangent-Lengths Theorem)

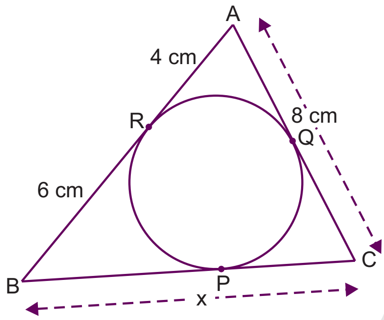
$\Rightarrow$ PA = PB = 10 cm

Also, CA = CQ (By Equal Tangent-Lengths Theorem)

$\Rightarrow$ CA = CQ = 2 cm ($\because$ CQ = 2 cm)

Now, PC = PA – CA = 10 – 2 = 8 cm

Example 8: *In the figure, the side BC, CA and AB of a triangle ABC touch the inscribed circle at P, Q and R respectively. Find the value of x.*



Solution: Since tangents from an external point to a circle are equal in length, therefore,

∴ AQ = AR (Tangents from A)

⇒ AQ = 4 cm

and BP = BR (Tangents from B)

⇒ BP = 6 cm

∴ CQ = CA – AQ = 8 cm – 4 cm = 4 cm

Now, CP = CQ (Tangents from C)

⇒ CP = 4 cm

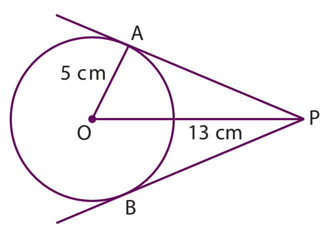
Now, BC = BP + CP = 6 cm + 4 cm

= 10 cm

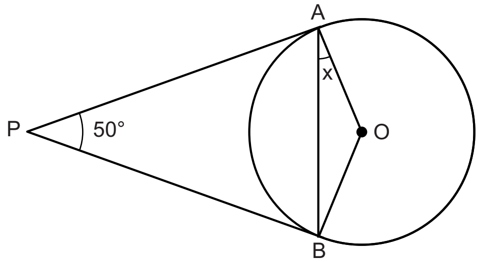
⇒ x = 10 cm.

EXERCISE 6.3

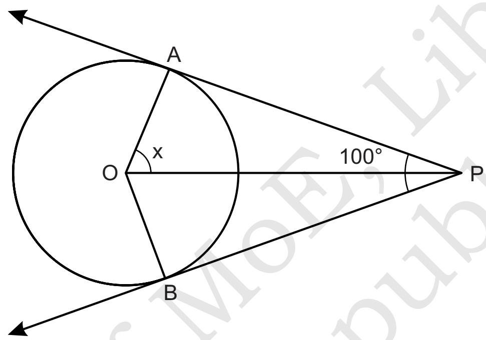
1. In the figure, PA and PB are two tangents to the circle with centre O, find their lengths.



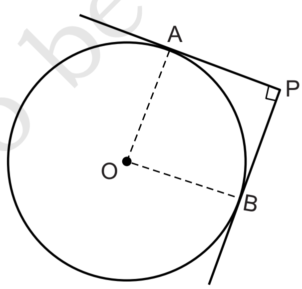
2. In the figure, PA and PB are two tangents to the circle with centre O such that ∠APB = 50°. Find the value of x.



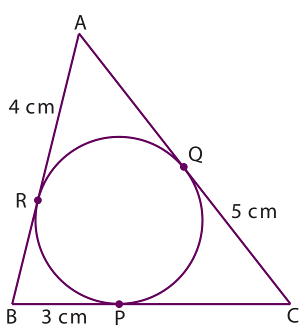
3. Two tangents PA and PB are drawn from an external point P to a circle with centre O as shown in figure. If they are inclined to each other at angle 100°, then what is the value of ∠POA = x ?



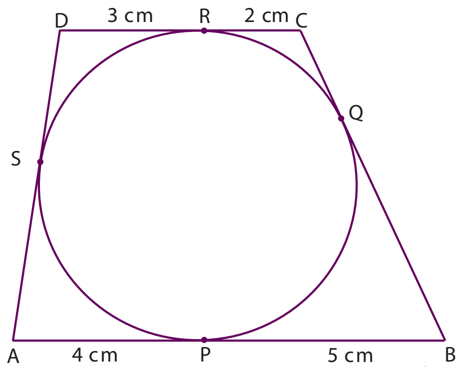
4. In the figure, PA and PB are two tangents drawn from an external point P to a circle with centre O and radius 4 cm. If PA ⊥ PB, then find the length of each tangent.



5. Find the perimeter of triangle ABC as shown in figure.

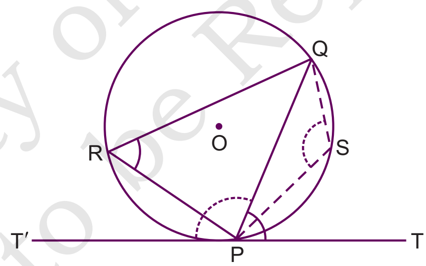


6. Find the perimeter of quadrilateral ABCD as shown in figure.



6.4. ALTERNATE SEGMENTS

Consider a circle with centre O. Any chord PQ divides the circle into two segments, PRQ and PSQ. Let T′PT be the tangent at P. The segment PRQ (on left of PQ) is called **alternate segment** to ∠QPT (on right of PQ) and segment PSQ (on right of PQ) is called **alternate segment** to ∠QPT′ (on left of PQ). Measure ∠QPT and ∠PRQ in the alternate segment. Are they equal? Measure ∠QPT′ and ∠PSQ in the alternate segment (see figure below). Are they equal?



The following theorem confirms the equality of these pairs angles.

Theorem 11: (Alternate Segment Theorem).

If a line touches a circle and from the point of contact, a chord is drawn, then the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

OR

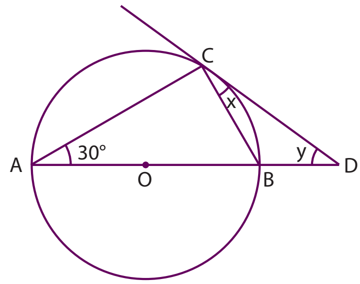
The angle between a chord and a tangent at the end of a chord is equal to the angle in the alternate segment (i.e., the angle in the other segment, not the one in which the first angle lies.

Thus, in the above figure,

$$\angle QPT = \angle PRQ \qquad \text{(Angle in alternate segments)}$$

and $$\angle QPT' = \angle PSQ \qquad \text{(Angle in alternate segment)}$$

Example 1: *In the figure, AB is a diameter and AC is a chord of the circle such that* $\angle BAC = 30°$. *Find x and y.*



Solution. $\angle BCD = \angle BAC$ (Angles in alternate segments)

$\Rightarrow \quad x = 30°$

Now, $\angle ACB = 90°$ (Angle in a semicircle)

$\Rightarrow \quad \angle BAC + \angle ABC = 90°$

$\Rightarrow \quad 30° + \angle ABC = 90°$

$\Rightarrow \quad \angle ABC = 90° - 30° = 60°$

Since, $\angle ABC + \angle DBC = 180°$ (Linear pair at B)

$\Rightarrow \quad 60° + \angle DBC = 180°$

$\Rightarrow \quad \angle DBC = 180° - 60° = 120°$

In triangle BCD,

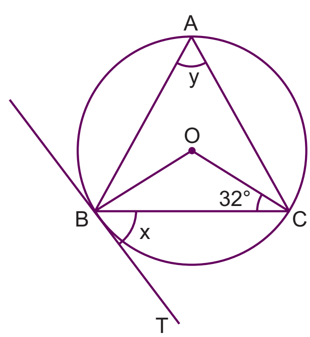
$\angle BCD + \angle DBC + \angle BDC = 180°$ (Sum of angles of a triangle)

$\Rightarrow \quad x + 120° + y = 180°$

$\Rightarrow \quad 30° + 120° + y = 180°$

$\Rightarrow \quad y = 180° - 150° = 30°.$

Example 2: *In the figure, BC is a chord of the circle with centre O and BT is the tangent to the circle at B. If* $\angle OCB = 32°$, *find x and y.*



Solution: $x = y$ (Angles in alternate segments)

In ΔOBC, $OB = OC$ (each = radius)

$\Rightarrow$ $\angle OBC = \angle OCB = 32°$

(Angles opposite to equal sides of a triangle are equal)

Also, $\angle OBC + \angle OCB + \angle BOC = 180°$

(Sum of all angles of a triangle is 180°)

$\Rightarrow$ $32° + 32° + \angle BOC = 180°$

$\Rightarrow$ $\angle BOC = 180° - 64° = 116°$

$\Rightarrow$ $2y = 116°$

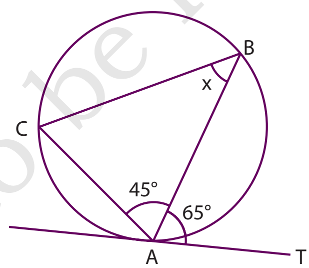
(Angle subtended by arc BC at the centre is double the angle it subtends at any point on the remaining circle)

$\Rightarrow$ $y = \frac{1}{2} \times 116° = 58°$

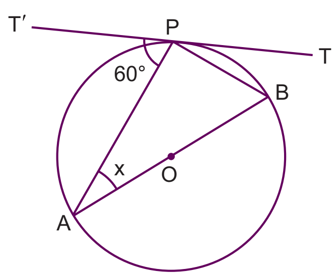
Hence, $x = y = 58°$

EXERCISE 6.4

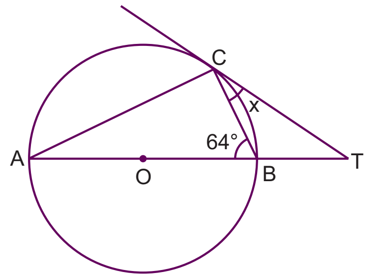
1. In the figure, AT is tangent to the circle at A. If AB and AC are two chords such that $\angle BAT = 65°$ and $\angle BAC = 45°$. Find x.



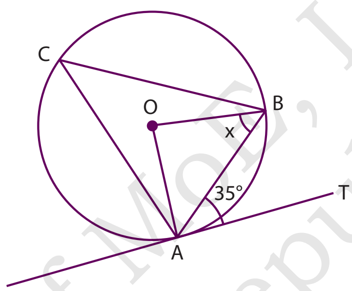
2. In the figure, AOB is a diameter and T′PT is the tangent at P. Find x, y and z.



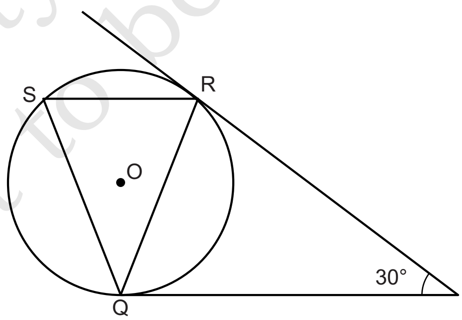
3. In the figure, AB is a diameter. The tangent at C meets AB produced at T. If ∠ABC = 64°. Find *x*.



4. In the figure, AB is a chord of circle with centre O and AT is the tangent at A. If ∠BAT = 35°. Find *x*.



5. In the figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find the ∠RQS.

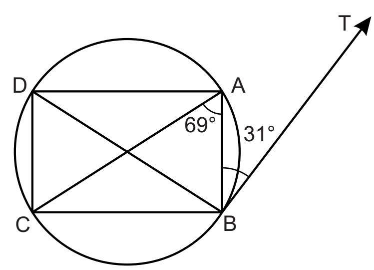


6. In the figure, TB touched the circle at B and BD is the diameter. Find

(i) ∠ADC

(ii) ∠ABC

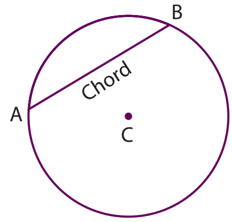
(iii) ∠CAD



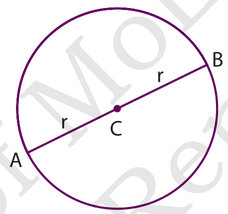
6.5. CIRCLE'S ARCS AND SECTORS

Given a circle with centre C and radius *r*, then

- A line segment joining any two distinct points on a circle is called a **chord** of the circle. Every circle has an infinite number of chords. Thus, line segment AB is the chord the circle.

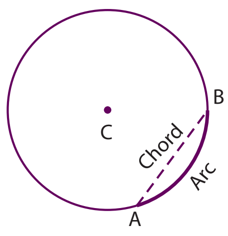


- A chord of a circle passing through its centre is called a **diameter.** Every circle has an infinite number of diameters. All diameters of a circle are equal in length. Thus, the chord AB is the diameter.



The length of (any) diameter = 2 × radius = $2r$

- A (continuous) part of a circle is called an **arc** of the circle. In the figure given below, AB is an arc of a circle with centre C. In symbols, arc AB is denoted by $\overset{\frown}{AB}$.



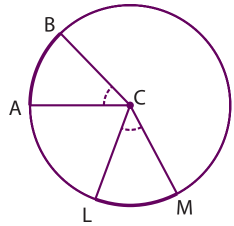
The line segment joining the two end points of arc AB is chord AB. The whole arc of a circle is called the circumference of the circle. In other words, the **circumference** of a circle is its boundary *i.e.*, the whole circular part of a circle.

The circumference of a circle = $2\pi r$ (when radius '*r*' is given)

= πd (when diameter '*d*' is given)

- Equal arcs of a circle subtend equal angles at the centre and vice versa. In the figure given below:

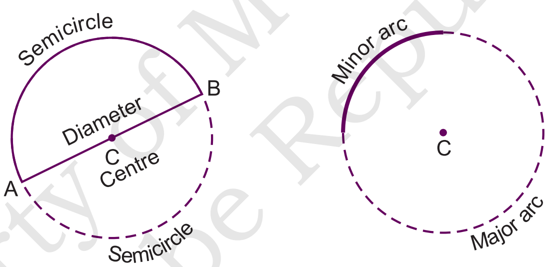
$$\overset{\frown}{AB} = \overset{\frown}{LM} \Leftrightarrow \angle ACB = \angle LCM$$



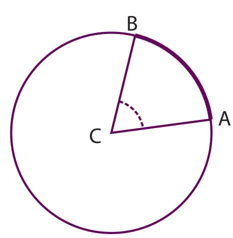
If the arcs are doubled, the central angles are also doubled. If the arcs are halved, the central angles are also halved.

In general, **angles at the centre of a circle are in the ratio of arcs subtending them.**

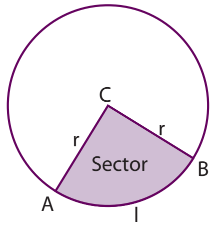
- Every diameter divides the circle into two equal parts, called **semicircles**. An arc less than a semicircle is called a **minor arc** and an arc greater than a semicircle is called a **major arc**.



- If AB is an arc of a circle with centre C, then ∠ACB (*i.e.*, the angle between the two radii CA and CB) is called the **angle subtended by arc AB at the centre C**.

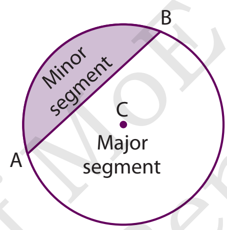


- The region enclosed by an arc of a circle and its two bounding radii is called a **sector of the circle**. In the figure given below, the shaded region ACB enclosed by arc AB and its two bounding radii CA and CB is a sector of the circle.

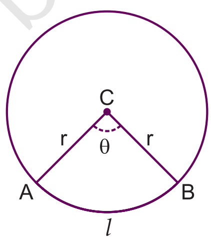


$$\text{Perimeter of sector ACB} = \overset{\frown}{AB} + CA + CB$$
$$= l + r + r$$
$$= l + 2r$$

- A chord of a circle divides the region enclosed by the circle into two parts. Each part is called a **segment**. The part containing the minor arc is called the **minor segment** and the part containing the major arc is called the **major segment**.



Theorem 12: *The length of an arc of a circle of radius r is* $\frac{\theta}{360°} \times 2\pi r$, *where θ is the angle subtended by the arc at the centre of the circle.*



In the figure, $l = \frac{\theta}{360°} \times 2\pi r$. (When θ in degree)

Proof. We know that the whole circle subtends an angle 360° at the centre. Also, the angles at the centre of a circle are in the ratio of the arcs subtending them. Therefore,

$$\frac{\angle ACB}{360°} = \frac{\text{acr AB}}{\text{Circinference of circle}}$$

$$\Rightarrow \quad \frac{\theta}{360°} = \frac{l}{2\pi r}$$

$$\Rightarrow \quad \frac{\theta}{360°} \times 2\pi r = l$$

$$\Rightarrow \quad l = \frac{\theta}{360°} \times 2\pi r$$

Note: l and r are lengths and, therefore, have same units of length.

Example 1: *An arc subtends an angle of 60° at the centre of a circle of radius 9 cm. Find the length of the arc.*

Solution: Here, $\theta = 60°$, $r = 9$ cm

Therefore, $l = \frac{\theta}{360°} \times 2\pi r = \frac{60°}{360°} \times 2\pi \times 9$

$= 3\pi = 9.42$ cm. (Using Calculator)

Example 2: *If in two circles, arcs of the same length subtend angles 60° and 75° at the respective centres, find the ratio of their radii.*

Solution. Let r_1 and r_2 be the radii of the two circles.

Given: $\theta_1 = 60°$ and $\theta_2 = 75°$

Let l be the length of each arc. Then

$$l = \frac{\theta_1}{360°} \times 2\pi r_1 = \frac{\theta_2}{360°} \times 2\pi r_2$$

$$\Rightarrow \quad r_1\theta_1 = r_2\theta_2$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{75°}{60°} = \frac{5}{4}$$

Hence, $r_1 : r_2 = 5 : 4$

Example 3: *The large hand of a clock is 42 cm long. How many centimetres does its extremity move in 20 minutes?* (use $\pi = \frac{22}{7}$)

Solution: In 60 minutes, the large hand (*i.e.*, minute hand) of a clock turns through 360°.

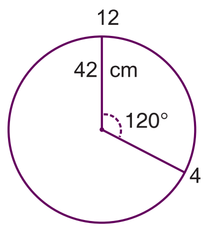
In 20 minutes, it turns through $\frac{360°}{60} \times 20 = 120°$

Now, $\theta = 120°$, $r = 42$ cm

$$\therefore \quad l = \frac{\theta}{360°} \times 2\pi r$$

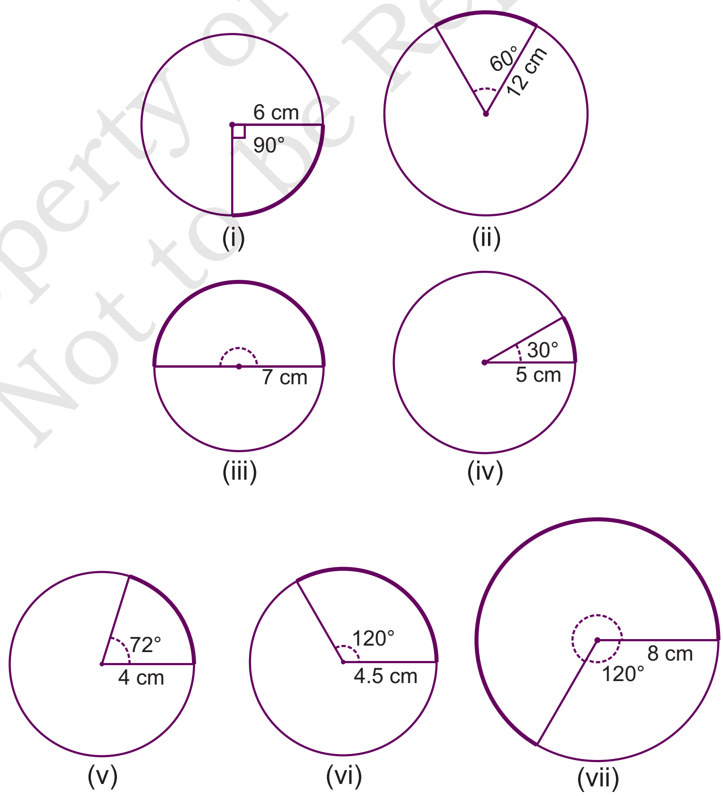
$$= \frac{120°}{360°} \times 2 \times \frac{22}{7} \times 42$$

$$= \frac{1}{3} \times 44 \times 6 = 88 \text{ cm}$$



EXERCISE 6.5

1. In each of the following figures, find the length of the arc shown thick: (Take $\pi = \frac{22}{7}$)



2. The minute hand of a clock is 1.5 cm long. How far does its tip move in 15 minutes?
3. A horse is tethered to a stake by a rope 30 m long. If the horse moves along the circumference of a circle always keeping the rope tight, find the distance travelled by the horse when the rope has traced out an angle of 105°.

Perimeter of a Circle

The circumference of the circle (*i.e.*, its circular part) is called its **perimeter**. In other words, the distance covered by travelling once around a circle is its perimeter, usually called its circumference.

Circumference of circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi'). In other words,

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

$$\Rightarrow \quad \text{Circumference} = \pi \times d$$

(where d is diameter of the circle)

$$\Rightarrow \quad = \pi \times 2r$$

($\because$ $d = 2r$, where r is radius of the circle)

$$= 2\pi r$$

$\therefore$ Perimeter/circumference of the circle = $2\pi r = \pi d$

For practical purpose, the value of π is taken as $\frac{22}{7}$ or 3.14.

Area of a Circle

Area of a circle = πr^2 ...(where r = radius of circle)

Area of a circle = $\frac{1}{4}\pi d^2$ ...(where d = diameter of circle)

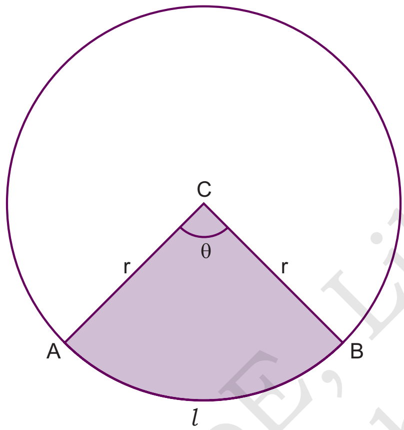
(**Note:** Diameter = 2 × radius or $d = 2r$).

Area of a Sector and Area of a Segment of a Circle

Now let us find area of a minor sector and area of a segment. Let ACB be a minor sector of a circle with centre C and radius r. If the arc AB substends an angle θ at centre C, then θ is called the **sector angle** of the sector ACB.

By geometry, $\dfrac{\text{Area of sector ACB}}{\text{Area of circle}} = \dfrac{\theta}{360^\circ}$

$\Rightarrow \quad \dfrac{\text{Area of sector ACB}}{\pi r^2} = \dfrac{\theta}{360^\circ}$



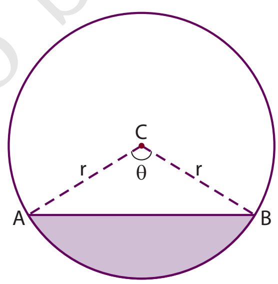
$\Rightarrow \quad \text{Area of sector ACB} = \dfrac{\theta}{360^\circ} \times \pi r^2$

Now, consider a minor segment of a circle with centre C and radius r.

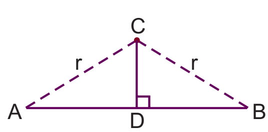
Area of segment (shaded)

= **Area of minor sector ACB – Area of isosceles triangle ACB**

$= \dfrac{\theta}{360^\circ} \times \pi r^2 - \dfrac{1}{2} r^2 \sin\theta$



Note: Draw CD perpendicular on AB, then D is mid-point of AB. If AB = $2a$, then AD = a.



By Pythagoras Theorem, in right angled triangle ADC, we have

$$CD^2 + AD^2 = AC^2$$

$$\Rightarrow \quad CD^2 + a^2 = r^2$$

$$\Rightarrow \quad CD^2 = r^2 - a^2$$

$$\Rightarrow \quad CD = r^2 - a^2$$

$$\text{Area of triangle ACB} = \frac{1}{2} \times AB \times CD$$

$$= \frac{1}{2}(2a)\sqrt{r^2 - a^2}$$

$$= a\sqrt{r^2 - a^2}$$

Example 4: *Find the circumference and area of a circle of radius 14 cm.*

Solution: We know that the circumference C and area A of a circle of radius r are given by C = $2\pi r$ and A = πr^2 respectively. Here, r = 8.4 cm.

$$\therefore \quad C = \text{Circumference} = 2\pi r = \frac{22}{7} \times 14 \text{ cm} = 88 \text{ cm}$$

$$A = \text{Area} = \pi r^2 = \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = 616 \text{ cm}^2$$

Example 5: *Find the area of a circle whose circumference is 22 cm. Also, find the area of a quadrant.*

Solution: Let r be the radius of the circle. It is given that the circumference of the circle is 22 cm.

$$\therefore \quad \text{Circumference} = 22 \text{ cm}$$

$$\Rightarrow \quad 2\pi r = 22$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 22 \quad \Rightarrow \quad r = \frac{7}{2} \text{ cm}$$

$$\therefore \quad \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2 = 38.5 \text{ cm}^2$$

$$\text{Area of a quadrant} = \frac{1}{4}\pi r^2 = \frac{1}{4} \times 38.5 \text{ cm}^2$$

$$= 9.625 \text{ cm}^2$$

Example 6: *Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.*

Solution: Let r be the radius of the circle whose circumference is equal to the sum of the circumferences of the circles of radii r_1 = 15 cm and r_2 = 8 cm. Then,

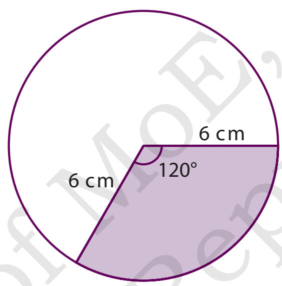
$$2\pi r = 2\pi r_1 + 2\pi r_2$$

$$\Rightarrow \quad 2\pi r = 2\pi(r_1 + r_2)$$

$$\Rightarrow \quad r = r_1 + r_2$$

$$\Rightarrow \quad r = (15 + 8) \text{ cm} = 33 \text{ cm.}$$

Example 7: *Find the area of the shaded region in the given figure.*



Solution: Shaded region is a sector of a circle.

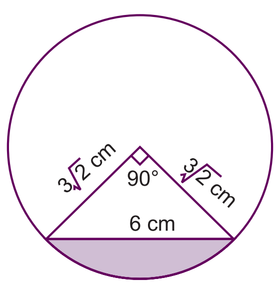
Here $\theta = 120°$, $r = 6$ cm

$$\therefore \quad \text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{120°}{360°} \times \pi \times 6^2$$

$$= 12\pi = 37.70 \text{ cm}^2.$$

Example 8. *Find the the area of the shaded region in the given figure.*



Solution: Shaded region is a segment of a circle.

Here $\theta = 90°$, $r = 3\sqrt{2}$ cm

$$\text{Area of shaded region} = \text{Area of sector} - \text{Area of triangle}$$

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{90°}{360°} \times \pi \times (3\sqrt{2})^2 - \frac{1}{2}(3\sqrt{2})^2 \sin 90°$$

$$= \frac{1}{4} \times \pi \times 18 - \frac{1}{2} \times 18 \times 1 = \frac{9\pi}{2} - 9$$

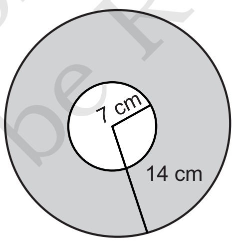
$$= 14.14 - 9 = 5.14 \text{ cm}^2$$

EXERCISE 6.6

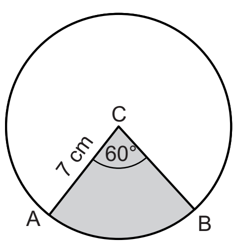
1. Find the area and circumference of a circle whose radius is 21 cm. (Take $\pi = 22/7$)

2. The area of a circle is 154 cm^2. Find its radius and circumference. Take $\pi = \frac{22}{7}$.

3. Find the area of the shaded region of the figure below.



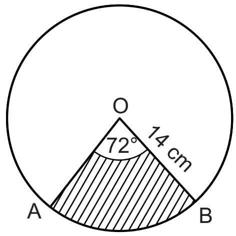
4. Find the area of sector CAB given the angle ACB is 60° and the radius of the circle is 7 cm.



5. The figure below shows a circle, centre O and radius 14 cm. The shaded region AOB is a sector with angle AOB = 72°. Find

(i) The length of the minor arc AB

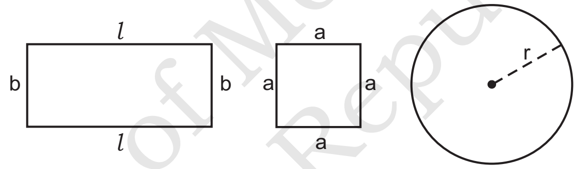
(ii) The area of the shaded sector AOB $\left[\text{Take } \pi = \frac{22}{7}\right]$



6.6. PERIMETER OF PLANE SHAPES/FIGURES

The perimeter of a closed **plane shape/figure** is the length of its boundary. The unit of measurement of the perimeter is the same as that of length.

In previous classes, we have learnt the perimeters of a rectangle, a square and a circle.



(a) If l and b are the length and breadth of a rectangle, then

$$\text{Perimeter of rectangle} = l + b + l + b = 2l + 2b = 2(l + b)$$

⇒ **Perimeter of rectangle = 2 (length + breadth)**

(b) If each side of a square is of length a, then

$$\textit{Perimeter of square} = a + a + a + a = 4a$$

⇒ **Perimeter of a square = 4 × length of a side.**

In general, if all sides of a closed plane figure are equal, then perimeter of closed plane figure = number of sides × length of a side. Thus,

(i) An **equilateral triangle** has 3 equal sides.

⇒ Perimeter of an equilateral triangle = 3 × length of a side

(ii) A **regular pentagon** has 5 equal sides.

⇒ Perimeter of a regular pentagon = 5 × length of a side

(iii) A **regular hexagon** has 6 equal sides.

⇒ Perimeter of a regular hexagon = 6 × length of a side

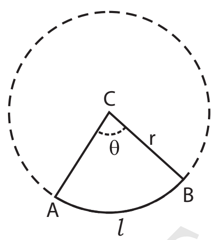
(iv) A **regular polygon** of n sides has n equal sides.

$\Rightarrow$ Perimeter of a regular polygon of n sides = $n \times$ length of a side

(c) Perimeter or circumference of a circle of radius $r = 2\pi r$.

(d) Now consider a sector ACB of a circle with centre C and radius r. Let arc AB = l and let $\angle$ACB = θ. Then we know that

$$l = \frac{\theta}{360°} \times 2\pi r$$



$\therefore$ Perimeter of sector ACB = arc AB + BC + CA = $l + r + r$

$$= \frac{\theta}{360°} \times 2\pi r + 2r$$

$$\Rightarrow \quad \text{Perimeter of a sector} = \frac{\theta}{360°} \times 2\pi r + 2r.$$

Example 1: *Find the perimeter of a rectangle whose length is 10 cm and breadth 8 cm.*

Solution: Here, l = 10 cm and b = 8 cm

$\therefore$ Perimeter of a rectangle = $2(l + b)$

$= 2(10 + 8) = 2 \times 18 = 36$ cm

Example 2: *The perimeter of a rectangle is 52 cm, its breadth is 6 cm. Find its length.*

Solution: Here, b = 6 cm, l = ?

$\therefore$ Perimeter of a rectangle = 52 cm

$\Rightarrow$ $2(l + b) = 52$ $\Rightarrow$ $2(l + 6) = 52$

$\Rightarrow$ $l + 6 = 26$ $\Rightarrow$ $l = 26 - 6 = 20$ cm

Example 3: *Find the perimeter of a square of side 9 cm.*

Solution: Here, Length of side, a = 9 cm

$\therefore$ Perimeter of a square = $4a = 4 \times 9 = 36$ cm

Example 4: *The perimeter of square is 32 cm. Find its length of side.*

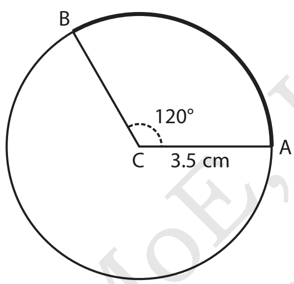
Solution: Let a be the length of a side of a square.

$\therefore$ Perimeter of a square $= 4a$

$\Rightarrow$ $32 = 4a$

$\Rightarrow$ $a = \frac{32}{4} = 8$ cm

Example 5: *Find the perimeter of the sector ACB in the figure.*



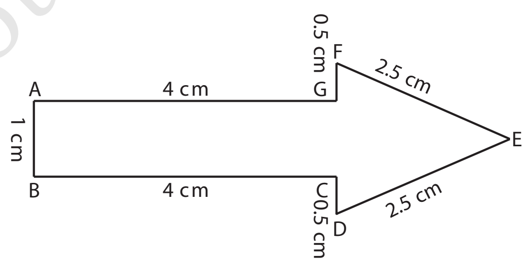
Solution: Here $\theta = 120°$, $r = 3.5$ cm

Therefore, perimeter of sector ACB

$$= \frac{\theta}{360°} \times 2\pi r + 2r = \frac{120°}{360°} \times 2\pi \times 3.5 + 2 \times 3.5$$

$$= \frac{7\pi}{3} + 7 = 7.33 + 7 = 14.33 \text{ cm.}$$

Example 6: *Find the perimeter of the following figure:*



Solution: Required perimeter = AB + BC + CD + DE + EF + FG + GA

(Start from vertex A, go around the plane figure and come back to A.)

= 1 + 4 + 0.5 + 2.5 + 2.5 + 0.5 + 4 = 15 cm.

EXERCISE 6.7

1. Find the perimeter of a rectangle whose length and breadth are 20 cm and 15 cm respectively.

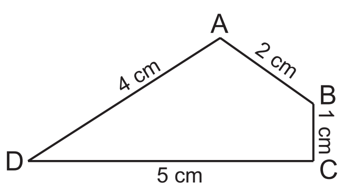
2. The perimeter of a rectangle 128 cm and its length is 38 cm.

3. The perimeter of a square is 108 cm. Find side of the square.

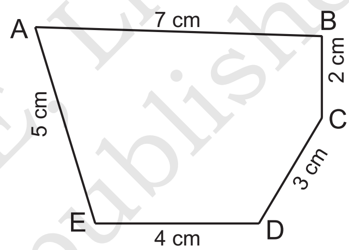
4. Find the perimeter of a quadrant of a circle of radius 7 cm.

$$\left[\text{Take } \pi = \frac{22}{7}\right]$$

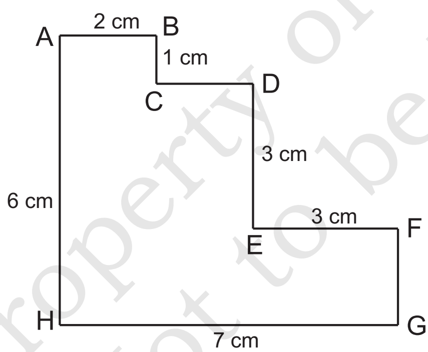
5. Find the perimeter of each of the following figures:



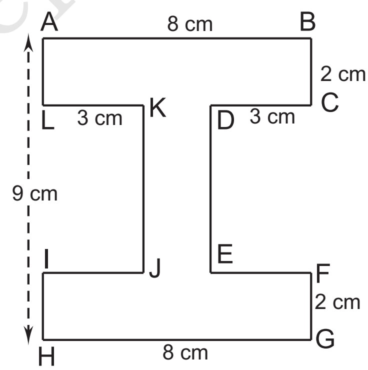
(i)



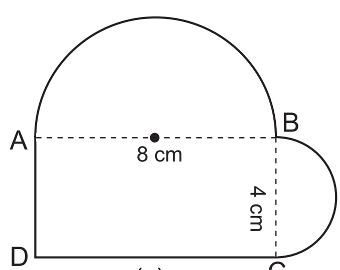
(ii)



(iii)



(iv)



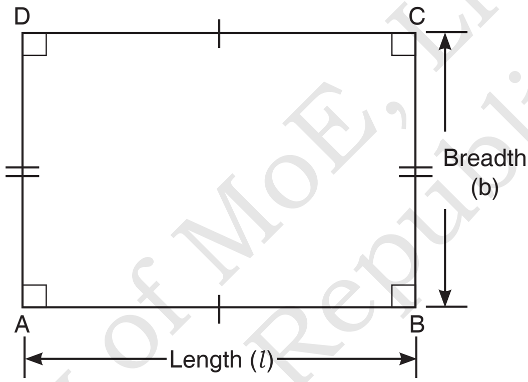
(v)

6.7. AREA OF RECTANGLES AND SQUARE

The **area** of closed plane figure is the measure of the region enclosed by its boundary. We are familiar with the areas of a rectangle, a square, a parallelogram and a triangle.

Rectangle

A rectangle is a four sided closed figure with all its sides are at right angles (90°) to each other. In rectangle, the opposite sides of a rectangle are equal and parallel to each other and all the angles of a rectangle are equal to 90°. Observe the rectangle given below to see its shape, sides and angles.



In a rectangle, AB = DC = length (l) and BC = AD = breadth (b).

Area of a rectangle (A) = length × breadth = $l \times b$

Properties of a Rectangle

Some of the important properties of a rectangle are given below.

- A rectangle is a **quadrilateral**. Since the sides of a rectangle are parallel, it is also called a **parallelogram**.
- The opposite sides of a rectangle are equal and parallel to each other.
- The interior angle of a rectangle at each vertex is 90°.
- The sum of all interior angles is 360°.
- The diagonals bisect each other.
- The length of the diagonals is equal.
- The length of the diagonals can be obtained using the Pythagoras theorem. The length of the diagonal with sides l and b is, diagonal $= \sqrt{(l^2 + b^2)}$.

Example 1. *The length and breadth of a rectangle is 12 cm units and 8 cm respectively. Find the area of the rectangle.*

Solution. Here, length, $l = 12$ cm; breadth, $b = 8$ cm

$$\therefore \quad \text{Area of a rectangle} = l \times b = 12 \times 8 = 96 \text{ cm}^2$$

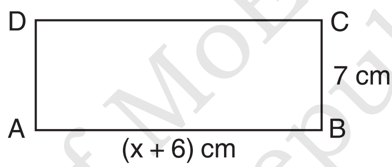
Example 2. *The breadth of a rectangular board is found to be 20 cm. Its area is found to be 220 cm^2. Find its length.*

Solution: Here, length, l = ?; breadth, $b = 20$ cm, Area of rectangle = 220 cm^2

$$\therefore \quad \text{Area of a rectangle} = l \times b \quad \Rightarrow \quad 220 = l \times 20$$

$$\Rightarrow \quad l = \frac{220}{20} = 11 \text{ cm}$$

Example 3: *In the figure, area of the rectangle is 154 cm^2. Find the value of x.*



Solution: Here, length, $l = (x + 6)$ cm; breadth, $b = 7$ cm, Area of rectangle = 154 cm^2

$$\therefore \quad \text{Area of a rectangle} = l \times b = (x + 6) \times 7$$

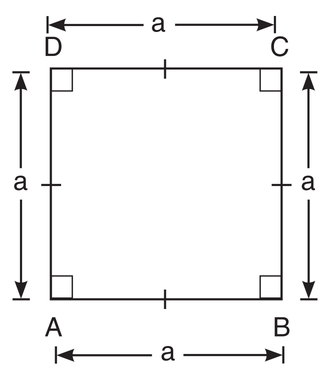
$$\Rightarrow \quad 154 = 7x + 42$$

$$\Rightarrow \quad 7x = 154 - 42 = 112$$

$$\Rightarrow \quad x = \frac{112}{7} = 16 \text{ cm}$$

Square

A square is a special rectangle in which all four sides are equal.



In a square, Length of each side of square = AB = BC = CD = DA = a.

Area of a square (A) = (length of side)^2 = a^2

Properties of a Square

Some of the important properties of a square are given below.

- All four interior angles are equal to 90°.
- All four sides of the square are congruent or equal to each other.
- The opposite sides of the square are parallel to each other.
- The diagonals of the square bisect each other at 90°.
- The two diagonals of the square are equal to each other.
- The diagonal of the square divide it into two similar isosceles triangles.
- The length of the diagonals can be obtained using the Pythagoras theorem. The length of the diagonal with side a is, diagonal = $\sqrt{(a^2 + a^2)} = \sqrt{2}\,a$

Example 4: *Let a square have side equal to 14 cm. Find out its area, perimeter and length of diagonal. (Take $\sqrt{2}$ = 1.414)*

Solution: Given: side of the square, a = 14 cm

$\therefore$ Area of a square = $a^2 = 14^2 = 196$ cm^2

And Perimeter of the square = $4a = 4 \times 14$ cm = 56 cm

Also, Length of the diagonal of a square

$= \sqrt{2}\,a = \sqrt{2} \times 14$

$= 14 \times 1.414 = 16.016$ cm

Example 5: *If area of a square is 289 cm^2, then what is the length of its sides? Also, find the perimeter of square.*

Solution: Given: Area of square, A = 289 cm^2

Let a be the length of the side a square.

$\therefore$ Area of a square, A = a^2

$\Rightarrow$ $289 = a^2$

$\Rightarrow$ $a = \sqrt{289} = 17$ cm

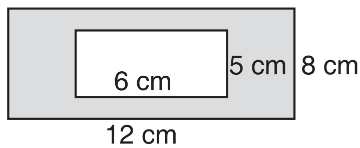
Now, Perimeter of the square = $4a = 4 \times 17$ cm = 68 cm

EXERCISE 6.8

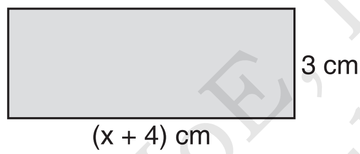
1. The length of a rectangle is 21 cm and its breadth is 12 cm. Find its perimeter and area.

2. The area of a rectangle is 154 cm^2, its breadth is 14 cm. Find its length.

3. Find the area of the shaded region in the figure below.



4. If the area of the figure below is 63 cm^2, find x.



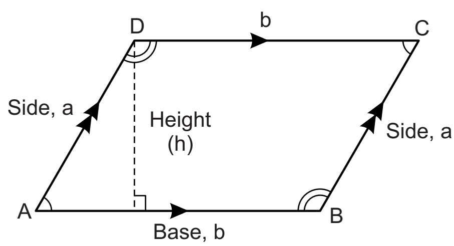
5. The perimeter of a rectangle is 48 cm and its breadth is 9 cm. Find its area.

6. Find the area and perimeter of a square of side 6 cm.

7. The area of a square is 196 cm^2. Find its perimeter.

6.8. AREA OF PARALLELOGRAMS

A parallelogram is a four sided closed figure (quadrilateral) in which both pairs of opposite (facing) sides are equal and parallel. Also, the opposite angles are of equal measure and the diagonals bisect each other, but sides are not at right angle (90°). The figure given below represents a parallelogram ABCD.



In a parallelogram ABCD, AB = DC and BC = AD. Also, AB ∥ DC and AD ∥ BC.

The area of parallelogram is calculated in two ways:

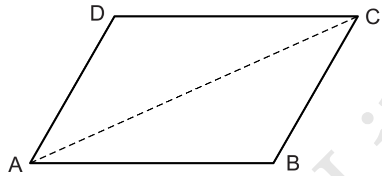
(*a*) When base (*b*) and perpendicular height (*h*) are given, then

Area of a parallelogram, A = Base × Perpendicular height

$= b \times h$

(b) When each diagonal divides the parallelogram into two triangles which are equal in area, then

Area of a parallelogram, A = 2 × Area of triangle ABC

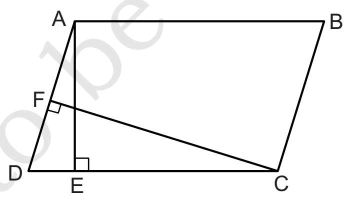


Example 1: *If the base of a parallelogram is equal to 12 cm and the height is 5 cm, then find its area.*

Solution: Given: length of base, b = 12 cm and height, h = 5 cm

∴ Area of a parallelogram, $A = b \times h = 12 \times 5 = 60 \text{ cm}^2$

Example 2: *In the figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.*



Solution: Area of parallelogram ABCD = Base × Height = AB × AE

$= 16 \times 8 \text{ cm}^2$

$= 128 \text{ cm}^2$ ...(1)

∴ Again, area of a parallelogram, ABCE = AD × CF

$= AD \times 10 \text{ cm}^2$ ...(2)

From (1) and (2), we get

$$AD \times 10 = 128$$

$$\Rightarrow AD = \frac{128}{10}$$

$$\Rightarrow AD = 12.8 \text{ cm}$$

Example 3: *The base of the parallelogram is thrice its height. If the area is 192 cm², find the base and height.*

Solution: Let the height of the parallelogram = h cm. Then, the base of the parallelogram = $3h$ cm.

$\therefore$ Area of a parallelogram, A = Base × Height

$\Rightarrow \quad 192 = 3h \times h$

$\Rightarrow \quad 192 = 3h^2$

$\Rightarrow \quad h^2 = 64$

$\Rightarrow \quad h = 8$ cm

The height of the parallelogram is 8 cm.

$\therefore$ Base of the parallelogram = $3h = 3 \times 8 = 24$ cm

EXERCISE 6.9

1. Find the area a parallelogram whose base is 14 cm and height is 11 cm.
2. The height of the parallelogram is twice its base. If the area is 512 cm², find the base and height.
3. The base of a parallelogram is $(x + 5)$ cm and the height is 4 cm. If the area of the parallelogram is 50 cm², find the value of x.
4. The perimeter of a parallelogram is 40 cm. If the height of the parallelogram is 5 cm and length of the adjacent side is 6 cm, find its area.
5. A parallelogram has sides equal to 10 cm and 8 cm. If the distance between the shortest sides is 5 cm, the find the distance between the longest of the parallelogram.

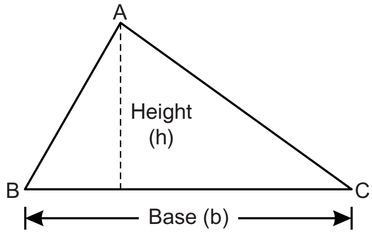
6.9. AREA OF TRIANGLES

A triangle is a three sided closed plane figure.

In general, the area of any triangle is determined by the following two ways:

(*a*) Area of a triangle when base (b) and height (h) are given.

$$\textbf{Area of a triangle, A} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}bh$$

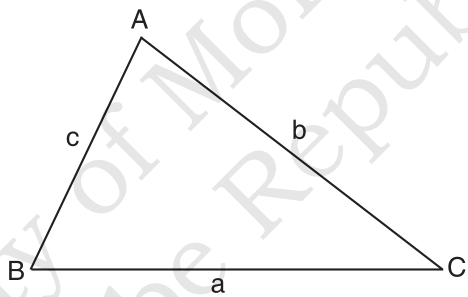


Note: Any side can be taken as base, then the height is the perpendicular distance of this side from the opposite vertex.

(*b*) Area of a triangle by **Heron's** (or **Hero's**) **formula:**

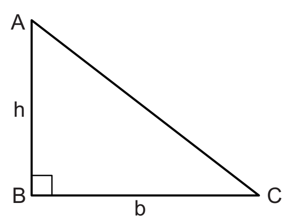
$$\textbf{Area of a triangle, A} = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b and c are the lengths of the sides of a triangle and s = semi-perimeter of triangle *i.e.*, half perimeter of a triangle $= \dfrac{a+b+c}{2}$



Area of Special Triangles

1. **Right angle triangle:** It is a triangle in which one angle is right angle or two sides are perpendicular.

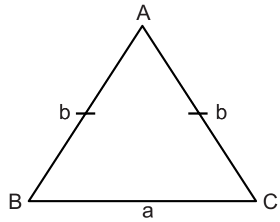


Area of a right angle triangle,

$$A = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}bh$$

$$= \frac{1}{2} \times \text{(Product of its legs containing the right angle)}$$

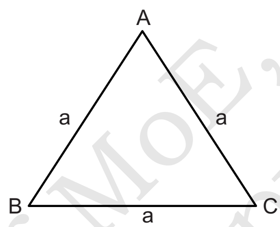
2. **Isosceles triangle:** It is a triangle whose two sides are equal.



Area of a isosceles triangle, A $= \frac{a}{4}\sqrt{4b^2 - a^2}$

where a is the length of base and b is the length of each equal side.

3. **Equilateral triangle:** It is a triangle whose sides are equal.



Area of a equilateral triangle, A $= \frac{\sqrt{3}}{4}a^2$

where a is the length of each side of a equilateral triangle.

Example 1: *Find the area a triangle whose base is 18 cm and height is 9 cm.*

Solution: Given: Length of base, b = 18 cm, Height, h = 9 cm

$\therefore$ Area of triangle, A $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 18 \times 9 = 81 \text{ cm}^2$

Example 2: *Find the height of the triangle whose base is two-thirds of its height and area is 225 cm^2.*

Solution: *Let the height of the triangle = h cm. Then, base of the triangle,*

$b = \frac{2h}{3}$ *cm.*

$\therefore$ Area of triangle, A = 225

$\Rightarrow \frac{1}{2} \times b \times h = 225$

$\Rightarrow \frac{1}{2} \times \frac{2h}{3} \times h = 225$

$$\Rightarrow \qquad h^2 = 225 \times 3$$

$$\Rightarrow \qquad h = \sqrt{225 \times 3} = 15\sqrt{3} \text{ cm}$$

Example 3: *Find the area of a triangle whose length of each side is 3 cm, 5 cm and 4 cm.*

Solution: Here, a = 3 cm, b = 5 cm and c = 4 cm.

Here, we shall use Heron's formula to find the area of the triangle.

$$\text{Semi-perimeter of triangle, } s = \frac{a+b+c}{2} = \frac{3+5+4}{2} = \frac{12}{2} = 6 \text{ cm}$$

$$\therefore \qquad \text{Area of triangle, A} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6(6-3)(6-5)(6-4)}$$

$$= \sqrt{6 \times 3 \times 1 \times 2} = \sqrt{36} = 6 \text{ cm}^2$$

Example 4: *The perimeter of an equilateral triangle is 60 cm. Find its area.*

Solution: Let the side of an equilateral triangle = a cm.

$\therefore$ Perimeter of equilateral triangle = 60

$$\Rightarrow \qquad 3a = 60$$

$$\Rightarrow \qquad a = 20 \text{ cm}$$

Now, area of equilateral triangle,

$$A = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 20^2 = 100\sqrt{3} \text{ cm}^2$$

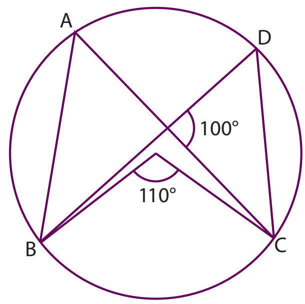
EXERCISE 6.10

1. Find the area a triangle whose base is 18 cm and height is 11 cm.
2. Find the height of a triangle whose base is 11 cm and its area is 176 cm^2.
3. Find the height of the triangle whose base is one-thirds of its height and area is 150 cm^2.
4. If the area of an equilateral triangle is $49\sqrt{3}$ cm^2, find its side.
5. Find the area of a triangle whose length of each side is 6 cm, 8 cm and 10 cm.

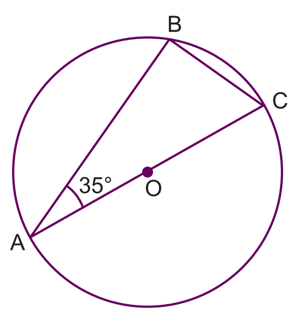
6. The base and the hypotenuse of a right angled triangle are 15 cm and 25 cm respectively. Find its area.
7. Perimeter of a triangle is 144 cm and the ratio of sides is 3 : 4 : 5. Find the area of the triangle.

MULTIPLE CHOICE QUESTIONS

1. Which one of the following is not true about circles?
   (a) Angles in the same segment of a circle are always equal.
   (b) Equal chords of a circle are always equidistant from its centre.
   (c) An angle in a semicircle is not always a right angle.
   (d) none of these.
2. Which one of the following is a correct statement?
   (a) A cyclic quadrilateral is one whose all vertices lie on a circle.
   (b) The sum of opposite angles of a cyclic quadrilateral is always 180°.
   (c) both (a) and (b)
   (d) none of these
3. In the figure, the measure of the angle ∠BDC is
   (a) 50°
   (b) 60°
   (c) 55°
   (d) 65°



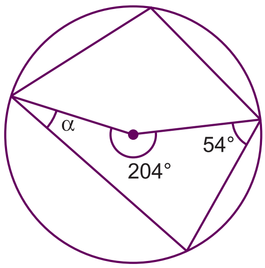
4. In the figure, which one of the following is true?



(a) $\angle ABC = 90°$ (b) $\angle ACB = 55°$

(c) both (a) and (b) (d) none of these

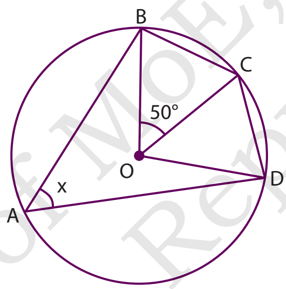
5. In the figure, the value of α is



(a) 54° (b) 204°

(c) 24° (d) 34°

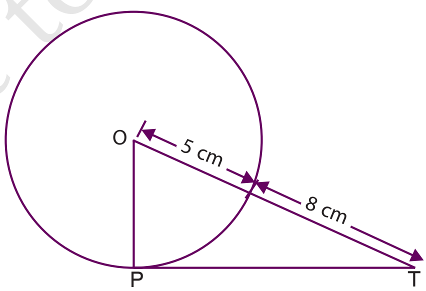
6. In the figure, if BC = CD, then, the value of x is



(a) 60° (b) 50°

(c) 70° (d) 40°

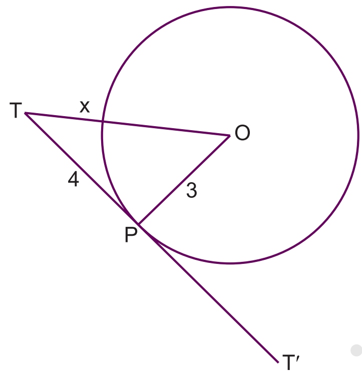
7. The length of the tangent to a circle as shown in the figure is



(a) 12 cm (b) 14 cm

(c) 8 cm (d) none of these

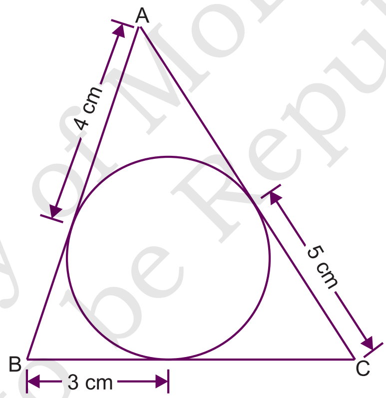
8. In the figure, if TPT′ represents a tangent to the given circle, then, the value of x is



(a) 4 (b) 3

(c) 6 (d) none of these

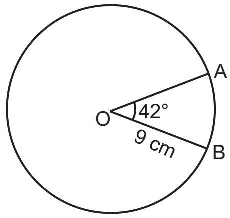
9. The perimeter of the triangle ABC shown in the figure is



(a) 24 cm (b) 14 cm

(c) 28 cm (d) 25 cm

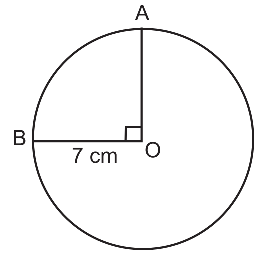
10. The figure shown below is of circle. The length of the minor arc is



(a) 13 cm (b) 14 cm

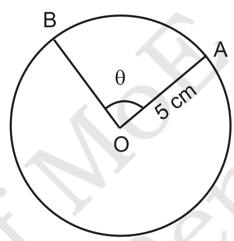
(c) 15 cm (d) none of these

11. A quadrant of a circle of radius 7 cm is shown in the figure. The perimeter of the quadrant OAB is



(a) 15 cm (b) 20 cm
(c) 25 cm (d) 30 cm

12. In the figure, the area of the sector AOB is 44 cm^2, the angle θ is

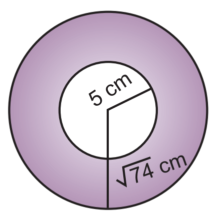


(a) 201° (b) 206°
(c) 201.6° (d) 206.1°

13. The area of a rectangular park is 2730 cm^2 and its width is 21 m. The length of the park is

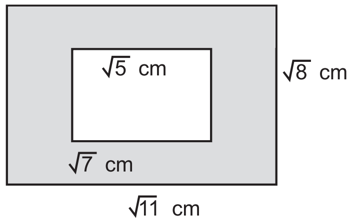
(a) 13 cm (b) 1.3 cm
(c) 30 cm (d) 1.3 m

14. The area A of the shaded portion in the figure is



(a) 154 cm^2 (b) 160 cm^2
(c) 174 cm^2 (d) none of these

15. The area A of the shaded portion as shown in the figure is



(a) 3.40 cm^2 (b) 3.60 cm^2

(c) 3.46 cm^2 (d) 3.66 cm^2